Homework 6

(*Due Friday, March 28, 2025*)

In these exercises, k denotes an algebraically closed field of characteristic zero.

1. (Lorenz 4.7.2) Let A be any algebra and B be any subalgebra of A. Let $B' = \{a \in A \mid ab = ba \, \forall b \in B\}$ denote the centralizer. Show that $B''' = B'$.

2. (Lorenz 4.4.1 on details for Spec(4)) Compute/verify the eigenvalues of the Jucys-Murphy elements on the Gelfand-Zetlin basis vectors of all irreducible representations of S_4 (see Example 4.10 in Lorenz).

3. Etingof 6.1.5 part (a) only: Suppose that Q is a a quiver of finite type (i.e. it has only finitely many indecomposable representations, up to isomorphism). Let (b_{ij}) be the number of edges between vertex i and vertex j in Q. Define

$$q(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} x_i^2 - \frac{1}{2} \sum_{i,j=1}^{n} b_{ij} x_i x_j$$

   Show that for any rational numbers $x_i \geq 0$ which are not all zero, we have $q(x_1, x_2, \ldots, x_n) > 0$. (Hint: Etingof's hint is useful. You may use Problem 6.1.2 without proof.)