

Homework 1

(Due Friday, January 31, 2025)

In these problems, assume \mathbb{k} is the ground field and $\otimes = \otimes_{\mathbb{k}}$.

1. If V_1 and V_2 are two vector spaces and U_i is a subspace of V_i for $i = 1, 2$, show that

$$(U_1 \otimes V_2) \cap (V_1 \otimes U_2) = U_1 \otimes U_2.$$

2. The *center* of an algebra A is $Z(A) = \{z \in A \mid za = az \forall a \in A\}$. Show that if A and B are algebras then $Z(A \otimes B) = Z(A) \otimes Z(B)$.
3. If A is any \mathbb{k} -algebra, let $M_n(A)$ denote the set of $n \times n$ -matrices with entries from A . It is again a \mathbb{k} -algebra with usual matrix operations (you don't need to show this). Prove that $A \otimes M_n(\mathbb{k}) \cong M_n(A)$ and $M_n(\mathbb{k}) \otimes M_m(\mathbb{k}) \cong M_{nm}(\mathbb{k})$.