MATH 510 Homework 9 **Name:**

- Due April 26 at the beginning of class.

1. Suppose that $A \in M_n$ is a real matrix whose n Gershgorin disks are all mutually disjoint. Show that all eigenvalues of A are real.

2. Suppose that $A \in M_n$ is idempotent ($A^2 = A$) but $A \neq I$. Show that A cannot be strictly diagonally dominant. (A is called *strictly diagonally dominant* if $|a_{ii}| > \sum_{\substack{1 \le j \le n \\ j \neq i}} |a_{ij}|$ for all $i = 1, 2, \dots, n$.)

3. Let $A \in M_n(\{0,1\})$ be a matrix whose entries are either 0 or 1. Show that A has property (SC) (equivalently, its directed graph $\Gamma(A)$ is strongly connected) if and only if every entry of the matrix $(I + A)^{n-1}$ is positive. (You may use that the i, j entry of A^k is the number of paths in $\Gamma(A)$ from i to j of length k.)

4. Let $A \in M_n$ be a skew-Hermitian matrix. For each $\varepsilon \in \mathbb{R}$, let $A_\varepsilon = \exp(\varepsilon A)$ and let λ_ε be an eigenvalue of A_ε. Show that
$$\limsup_{\varepsilon \to 0} \frac{1}{\varepsilon} |\lambda_\varepsilon - 1| \le \|A\|_\infty,$$
where $\|A\|_\infty = \max_p \sum_j |a_{pj}|$ is the max-row-sum matrix norm.

5. What are necessary and sufficient conditions on the signs of its minors for a Hermitian matrix A to be negative definite (semidefinite)? (*Hint:* This is related to Sylvester's Criterion.)