

Math 403/503 Spring 2024

Homework 9, due April 17

1. Let E/F be a field extension. If $\alpha, \beta, \gamma \in E$ are algebraic over F , explain why $\alpha + \beta + \gamma$ is algebraic over F .
2. Let E be a splitting field for an irreducible polynomial $p(x) \in F[x]$ over F , and α and β be two roots of $p(x)$. Show that there is an isomorphism $\Phi: E \rightarrow E$ such that $\Phi(\alpha) = \beta$.
3. Prove that the fields $\mathbb{Q}(\sqrt[4]{3})$ and $\mathbb{Q}(\sqrt[4]{3}i)$ are isomorphic but not equal.
4. Prove or disprove: $\mathbb{Q}(\sqrt{2}) \cong \mathbb{Q}(\sqrt{3})$.
5. Let K be an algebraic extension of E , and E an algebraic extension of F . Prove that K is algebraic over F . [Caution: Do not assume that the extensions are finite.]