

Math 403/503 Spring 2024

Homework 10, due April 24

1. Show that if $\alpha \in \mathbb{R}$ is a positive and constructible number, then $\sqrt{\alpha}$ is also constructible. (In other words, $\mathbb{R}_{>0} \cap \mathbb{R}_c$ is closed under square roots.)
2. Show that the regular 9-gon is not constructible with straightedge and compass (without actually performing the construction).
3. Can a cube be constructed with three times the volume of a given cube?
4. Show that if $A = (a_1, a_2), B = (b_1, b_2), C = (c_1, c_2), D = (d_1, d_2)$ are four distinct points in the plane, such that the lines \overline{AB} and \overline{CD} intersect at $Q = (q_1, q_2)$, show that q_1 and q_2 belong to the field $\mathbb{Q}(a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2)$.
5. Prove that the cosine of one degree ($\cos 1^\circ$) is algebraic over \mathbb{Q} but not constructible.