

# Math 403/503 Spring 2024

## Homework 1

1. Let  $(M, \cdot, e)$  be a monoid. Show that  $M^\times$  is a submonoid of  $M$ , and that  $M^\times$  is a group (using the same operation as in  $M$ ).
2. Determine for which sets  $X$  the monoid  $(X^X, \circ, \text{Id}_X)$  is commutative.
3. Let  $(M, \cdot, e)$  and  $(N, *, e')$  be monoids. Show that  $M \times N$  becomes a monoid in a natural way.
4. Let  $G = S_3$  and  $H = \{(1), (12)\}$ ,  $N = \{(1), (123), (132)\}$ .
  - (a) Is it true that  $gH = Hg$  for all  $g \in G$ ?
  - (b) Is it true that  $gN = Ng$  for all  $g \in G$ ? (Recall that  $gH = \{gh \mid h \in H\}$  and similarly for  $Hg$ .)