Math 365 – Sample Problems for Exam 1

Section 1.1 (The Complex Numbers) and Section 1.2 (Some Geometry).

1. Find
$$\mathrm{Re}\left(\frac{3+7i}{i-1}\right), \qquad (2+3i)^5, \qquad \mathrm{Arg}(-\sqrt{3}+i)$$
2. Find all complex numbers z such that $z^7 = 1-i$.

Section 1.3 (Subsets of the Plane).

3. Sketch the region D and determine the boundary of D. Is D open? connected? closed? convex? a domain?
$$\text{(a) } D=\{z : |z| > \mathrm{Re}\, z\} \qquad \text{(b) } D=\{z : -1 < \mathrm{Re}\, z < |\mathrm{Im}\, z| < 1\}$$
$$\text{(c) } D=\{z : |\mathrm{Re}\, z| \le 2,\ |\mathrm{Im}\, z| \le 2,\ |z| > 1\}$$

Section 1.4 (Functions and Limits).

4. Find the limit or explain why it doesn't exist.
$$\text{(a) } \lim_{n\to\infty}\left(\frac{2+i}{\sqrt{3}}\right)^n \qquad \text{(b) } \lim_{n\to\infty}\frac{n+i}{i-e^n}$$
5. Find the limit or explain why it doesn't exist.
$$\text{(a) } \lim_{z\to\infty} e^{3-iz} \qquad \text{(b) } \lim_{z\to 1+i}\frac{z^2-2i}{z-(1+i)}$$
6. Is the following series convergent? Why/Why not?
$$\text{(a) } \sum_{n=0}^{\infty} n^2\left(\frac{2}{3-i}\right)^n \qquad \text{(b) } \sum_{n=0}^{\infty}\frac{1}{2+i^n}$$

Section 1.5 (The Exponential, Logarithm, and Trig Functions).

7. Show that $|\sin(z)| \le e^{|\mathrm{Im}\, z|}$ for all complex numbers z.
8. Find all values for (a) $\log(1-i)$ (b) $(1+i)^i$.

Section 1.6 (Line Integrals and Green's Theorem).

9. Compute the line integral
$$\int_\gamma \left(z^2+\frac{1}{z}\right)dz$$
where γ is the circle of radius 2 centered at the origin and oriented counter-clockwise.
10. Evaluate the line integral
$$\int_\gamma (3z+i)\,dz$$
where γ is the rectangle with corners $1+i$, $-1+i$, $-1-i$, $1-i$, oriented counter-clockwise.