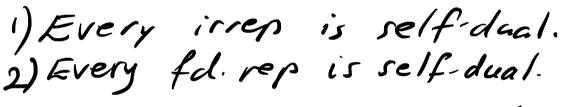
MATH 618 LECTURE 12

83.5.1 \$ 3.5.2

<u>HW12</u>: Prove that TFAE for a finite group G:



3) Every gEG is conjugate to its inverse.

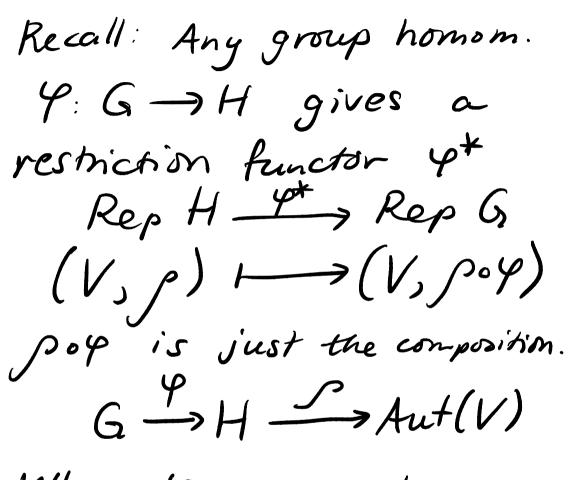
Examples For the groups G = (C2)ⁿ, G=Dn (cyclic grp dihedral $G = S_n$, every element is conjugate to its inverse, hence every fol (ir) rep is self-dual. Tensoring with 1-dim'l reps. If V is a 1-dim'l rep of a group G, and W is any irrep of G then VOW is also an irrep of G: The map {subreps of W} -> {subreps of VoW} $\mathcal{U} \longmapsto \mathcal{V} \otimes \mathcal{U}$ is a bijection.

Application to symmetric group: Def Let V be a fol rep of Sn. The corresponding sign-twisted rep is $V^{\perp} := V_{sgn} \otimes V$ Note Xy = Xsgn : Xv and thus $V^{\pm} \cong V$ iff for every odd $\sigma \in S_n$ we have $\mathcal{X}_V(\sigma) = 0$. $\frac{E\chi}{\chi_{triv}} \frac{S_3}{11} \frac{(1)}{11} \frac{(12)}{11} \frac{$

Representation ring The representation ring R(G) of a finite group G is the free abelian group on the set of equivalence classes [V] of fd reps of G modulo the subgroup generated by [V⊕W]-([V] +[W]) for all fd reps V, W. The multiplication in RG) is defined by $[V] \cdot [W] = [V \otimes W].$

Example Consider V2 &V2. Its character is $\begin{array}{c} \chi_{V_2\otimes V_2} = \chi_{V_2} \cdot \chi_{V_2} \quad \text{so its} \\ \text{values are:} \\ \hline \chi_{V_2\otimes V_2} \quad \begin{array}{c} (1) \quad (12) \quad (123) \\ \chi_{V_2\otimes V_2} \quad \begin{array}{c} 4 \quad 0 \quad 1 \end{array} \end{array}$ Note! $\chi_{V_2 \otimes V_2} = \chi_{triv} + \chi_{r_2} = \chi_{V_{triv} \oplus V_{sgn} \oplus V_2} = \chi_{V_{triv} \oplus V_{sgn} \oplus V_2}$ $= V_2 \otimes V_2 \cong V_{triv} \oplus V_{sgn} \oplus V_2$ $= V_{2} [V_2] \cdot [V_2] = [V_{triv}] t [V_{sgn}] t [V_2] in \mathcal{R}(S_3).$

Inflation



When φ is surjective we call this process inflation. Main point: If (V, p) is an irrep of H, then $(V, p \circ \varphi)$ is an irrep of G : (Pf: Exercise.)