Homework 1 – MATH 3010 Fall 2024

1. Exercise 1.2 from the book[1]

2. Exercise 1.3 from the book

3. Let $f : A \to X$ be any function. Define a relation $\sim$ on A by $a \sim b \Leftrightarrow f(a) = f(b)$. Prove that $\sim$ is an equivalence relation on A.

4. Describe the equivalence classes in the previous problem.

5. Find the image of the function f when
   (a) $f : \mathbb{Z} \to \mathbb{Q}$; $f(x) = x - 1$.
   (b) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = -x^2 + 1$.

6. Let $A = \{1,\ 2,\ 3,\ 4\}$. Exhibit functions f and g from A to A such that $f \circ g \neq g \circ f$.

7. Prove that the given function is injective.
   (a) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = x^3$.
   (b) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = -3x + 5$.

8. Prove that the given function is surjective.
   (a) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = -3x + 5$.
   (b) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$; $f(a, b) = a/b$ when $b \neq 0$ and 0 when $b = 0$.

9. (a) Let $f : B \to C$ and $g : C \to D$ be functions such that $g \circ f$ is injective. Prove that f is injective.
   (b) Give an example of the situation in part (a) in which g is not injective.

10. Let $2\mathbb{Z}$ denote the set of even integers. Let $\sim$ be a relation on $\mathbb{Q}$ defined by $x \sim y \Leftrightarrow x - y \in 2\mathbb{Z}$. Prove that $\sim$ is an equivalence relation on $\mathbb{Q}$.

[1] By "the book" we mean the course textbook *Algebra: Notes from the underground* by Aluffi.