Math 301 Homework 9 (due on November 1, 2023)

Section 5.2

2. Write out the addition and multiplication tables for the congruence-class ring $\mathbb{Z}_3[x]/(x^2+1)$.

6. Determine the rules for addition and multiplication of congruence classes in $\mathbb{Q}[x]/(x^2-2)$.

14. In each part explain why $[f(x)]$ is a unit in $\mathbf{F}[x]/(p(x))$ and find its inverse.

(a) $[f(x)] = [2x-3] \in \mathbb{Q}[x]/(x^2-2)$.

(b) $[f(x)] = [x^2+x+1] \in \mathbb{Z}_3[x]/(x^2+1)$.

16. Show that $\mathbb{Q}[x]/(x^2-2)$ is a field.

Section 5.3

1. Determine whether the given congruence-class ring is a field. Justify your answer.

(a) $\mathbb{Z}_3[x]/(x^3+2x^2+x+1)$

(b) $\mathbb{Z}_5[x]/(2x^3-4x^2+2x+1)$.

2.

(a) Verify that $\mathbb{Q}(\sqrt{2}) = \{r + s\sqrt{2}\ r, s \in \mathbb{Q}\}$ is a subfield of $\mathbb{R}$.

(b) Show that $\mathbb{Q}(\sqrt{2})$ is isomorphic to $\mathbb{Q}[x]/(x^2-2)$.

6. Let $p(x)$ be irreducible in $\mathbf{F}[x]$. If $[f(x)] \neq [0_\mathbf{F}]$ in $\mathbf{F}[x]/(p(x))$ and $h(x) \in \mathbf{F}[x]$, prove that there exists $g(x) \in \mathbf{F}[x]$ such that $[f(x)][g(x)] = [h(x)]$ in $\mathbf{F}[x]/(p(x))$. [*Hint :* Theorem 5.10 and Exercise 12 (b) in Section 3.2]

8. If $p(x)$ is an irreducible quadratic polynomial in $F[x]$, show that $F[x]/(p(x))$ contains all the roots of $p(x)$.