Math 301 Homework 8 (due on October 25, 2023)

Section 4.5

4. Show that each polynomial is irreducible in $\mathbb{Q}[x]$, as in Example 3.

(a) $f(x) = x^4 + 2x^3 + x + 1$.

(b) $f(x) = x^4 - 2x^2 + 8x + 1$

8. Give an example of a polynomial $f(x) \in \mathbb{Z}[x]$ and a prime p such that $f(x)$ is reducible in $\mathbb{Q}[x]$ but $\overline{f}(x)$ is irreducible in $\mathbb{Z}_p[x]$. Does this contradict Theorem 4.25?

18. Which of these polynomials are irreducible in $\mathbb{Q}[x]$?

(a) $f(x) = x^4 - x^2 + 1$.

(b) $f(x) = x^4 + x + 1$.

19(b). Write the polynomial $x^7 - 2x^6 - 6x^4 - 15x^2 - 33x - 9$ as a product of irreducible polynomials in $\mathbb{Q}[x]$.

Section 4.6

1 (b). Find all roots in $\mathbb{C}$ of $f(x) = x^4 - 2x^3 - x^2 + 6x - 6$; given $1 + i$ is a root.

4. Factor $x^2 + x + 1 + i$ in $\mathbb{C}[x]$.

Section 5.1

4. Show that, under congruence modulo $x^3 + 2x + 1$ in $\mathbb{Z}_3[x]$, there are exactly 27 distinct congruence classes.

8. Prove or disprove: If $p(x)$ is relatively prime to $k(x)$ and $f(x)k(x) \equiv g(x)k(x) \pmod{p(x)}$, then $f(x) \equiv g(x) \pmod{p(x)}$.

10. Prove or disprove: If $p(x)$ is irreducible in $\mathbf{F}[x]$ and $f(x)g(x) \equiv 0_{\mathbf{F}} \pmod{p(x)}$, then $f(x) \equiv 0_{\mathbf{F}} \pmod{p(x)}$ or $g(x) \equiv 0_{\mathbf{F}} \pmod{p(x)}$.

12. If $f(x)$ is relatively prime to $p(x)$, prove that there is a polynomial $g(x) \in \mathbf{F}[x]$ such that $f(x)g(x) \equiv 1_{\mathbf{F}} \pmod{p(x)}$.