Math 301 Homework 7 (due on October 18, 2023)

Section 4.3

6. Show that $x^2 + 1$ is irreducible in $\mathbb{Q}[x]$.

12. Express $x^4 - 4$ as a product of irreducibles in $\mathbb{Q}[x]$, $\mathbb{R}[x]$, and in $\mathbb{C}[x]$.

14. Show that $x^2 + x$ can be factored in two ways in $\mathbb{Z}_6[x]$ as the product of non-constant polynomials that are not units and not associates of x or $x + 1$.

22.

(a) Show that $x^3 + a$ is reducible in $\mathbb{Z}_3[x]$ for each $a \in \mathbb{Z}_5$.

(b) Show that $x^5 + a$ is reducible in $\mathbb{Z}_5[x]$ for each $a \in \mathbb{Z}_5$.

Section 4.4

2. Find the remainder when $f(x)$ is divided by $g(x)$:

(a) $f(x) = x^{10} + x^8$ and $g(x) = x - 1$ in $\mathbb{Q}[x]$.

(d) (Typo corrected on Oct 16, 11am) $f(x) = 2x^5 - 3x^4 + x^3 + \mathbf{2x} + 3$ and $g(x) = x - 3$ in $\mathbb{Z}_5[x]$.

8. Determine if the given polynomial is irreducible.

(c) $x^2 + 7$ in $\mathbb{C}[x]$.

(d) $2x^3 + x^2 + 2x + 2$ in $\mathbb{Z}_5[x]$.

(e) $x^3 - 9$ in $\mathbb{Z}_{11}[x]$.

(f) $x^4 + x^2 + 1$ in $\mathbb{Z}_3[x]$.

10. Find a prime $p > 5$ such that $x^2 + 1$ is reducible in $\mathbb{Z}_p[x]$.

16. Let $f(x), g(x) \in F[x]$ have degree $\leq n$ and let $c_0, \ldots, c_n$ be distinct elements of F. If $f(c_i) = g(c_i)$ for $i = 0, \ldots, n$, prove that $f(x) = g(x)$ in $F[x]$.