Math 301 Homework 4 (due on September 20, 2023)

Section 2.2

6. Solve the equation $x^2 \oplus [8] \odot x = [0]$ in $\mathbb{Z}_9$.

8. Solve the equation $x^3 \oplus x^2 = [2]$ in $\mathbb{Z}_{10}$.

12. Prove or disprove: If $[a] \odot [b] = [0]$ in $\mathbb{Z}_n$, then $[a] = [0]$ or $[b] = [0]$.

14(c). If p is a positive prime, show that the only solutions of $x^2 \oplus x = [0]$ in $\mathbb{Z}_p$ are $[0]$ and $[p-1]$.

16. Find all $[a]$ in $\mathbb{Z}_n$ for which the equation $[a] \odot x = [1]$ has a solution, in the case when

(a) $n = 5$, (b) $n = 4$, (c) $n = 3$, (d) $n = 6$

Section 2.3

2. Find all zero-divisors in

(a) $\mathbb{Z}_7$, (b) $\mathbb{Z}_8$, (c) $\mathbb{Z}_9$, (d) $\mathbb{Z}_{10}$

6. If n is composite, prove that there is at least one zero-divisor in $\mathbb{Z}_n$. (See Exercise 2.)

10. Prove that every nonzero element of $\mathbb{Z}_n$ is either a unit or a zero-divisor, but not both. [Hint: Exercise 9 provides the proof of "not both".]

Section 3.1

2. Let $R = \{0, e, b, c\}$ with addition and multiplication defined by the tables below. Assume associativity and distributivity and show that R is a ring with identity. Is R commutative? Is R a field?

+	0	e	b	c
0	0	e	b	c
e	e	0	c	b
b	b	c	0	e
c	c	b	e	0

·	0	e	b	c
0	0	0	0	0
e	0	e	b	c
b	0	b	b	0
c	0	c	0	c

4. Find matrices A and C in $M(\mathbb{R})$ such that $AC = \mathbf{0}$ but $CA \neq \mathbf{0}$, where $\mathbf{0}$ is the zero matrix. [Hint: Example 6.]