Math 301 Homework 2 (due on September 6, 2023)

Section 1.2

1. Find the greatest common divisor using Euclid's algorithm.

d) $(143, 231)$

h) $(12378, 3054)$

4.

a) If $a|b$ and $a|c$, prove that $a|(b+c)$.

b) If $a|b$ and $a|c$, prove that $a|(br+ct)$ for any $r, t \in \mathbb{Z}$.

22. If $(a, c) = 1$ and $(b, c) = 1$, prove that $(ab, c) = 1$.

24. Let $a, b, c \in \mathbb{Z}$. Prove that the equation $ax + by = c$ has integer solutions if and only if $(a, b)|c$.

Section 1.3

14. Let p be an integer other than $0, \pm 1$ with this property: Whenever b and c are integers such that $p|bc$, then $p|b$ or $p|c$. Prove that p is prime.
Hint: If d is a divisor of p, say $p = dt$, then $p|d$ or $p|t$. Show that this implies $d = \pm p$ or $d = \pm 1$.

22. Let $n = p_1^{r_1} p_2^{r_2} \cdots p_n^{r_n}$ where p_i are distinct primes and each $r_i > 0$. Prove that n is a perfect square if and only if each r_i is even.

30.

a) Prove that there are no nonzero integers a, b such that $a^2 = 2b^2$.
Hint: Use the Fundamental Theorem of Arithmetic.

b) Prove that $\sqrt{2}$ is irrational.
Hint: Use proof by contradiction (Appendix A). Assume that $\sqrt{2} = a/b$ (with $a, b \in \mathbb{Z}$) and use part (a) to reach a contradiction.

34. Prove or disprove: If n is an integer and $n > 2$, then there exists a prime p such that $n < p < n!$.

36. Let p, q be primes with $p \geq 5$, $q \geq 5$. Prove that $24|(p^2 - q^2)$.