Math 301 Homework 12 (due on December 6, 2023)

Section 7.5

3, 4. Express each permutation as a product of disjoint cycles **and** as a product of transpositions.

(b) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 5 & 1 & 2 & 4 & 6 & 8 & 9 & 7 \end{pmatrix}$

(d) (14)(27)(523)(34)(1472)

6. Find the order of each permutation.

(b) (123)(456).

(c) (123)(435).

(d) (1234)(4231).

(e) (1234)(24)(43215).

14 Show that $\beta = (1236)(5910)(465)(5678)$ has order 21 in S_n ($n \geq 10$).

21. Find the order of σ^{1000}, where σ is the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 7 & 8 & 9 & 4 & 5 & 2 & 1 & 6 \end{pmatrix}.$$

22. Show that S_{10} contains elements of orders 10, 20, 30. Does it contain an element of order 40?

Section 8.1

6. Let $K = \langle 3 \rangle$; $G = U_{32}$. List the distinct K cosets of K in G.

20. A group has fewer than 100 elements and subgroups of order 10 and 25. What is the order of the group?

26. Prove that a group of order 8 must contain an element of order 2.

29. Let H and K be subgroups of a finite group G such that $K \subseteq H$, $[G : H]$ is finite, and $[H : K]$ is finite Prove that $[G : K] = [G : H][H : K]$. [*Hint:* Lagrange.]

33.

(a) If a and b each have order 3 in a group and $a^2 = b^2$, prove that $a = b$. [*Hint:* What are a^{-1} and b^{-1}.]

(b) If G is a finite group, prove that there is an even number of elements of order 3 in G.