**Math 301 Homework 11 (due on November 29, 2023)**

## Section 7.3

**6.** Describe $\langle 2 \rangle$ in the multiplicative group of nonzero elements in $\mathbb{Z}_{11}$.

**12.** Show that the additive group $\mathbb{Z}_2 \times \mathbb{Z}_4$ is not cyclic but generated by two elements.

**14.** Let $H$ and $K$ be subgroups of a group $G$.

(a) Show by example that $H \cup K$ need not be a subgroup of $G$.

(b) Prove that $H \cup K$ is a subgroup of $G$ if and only if $H \subseteq K$ or $K \subseteq H$.

**28.** Let $G$ be an abelian group and $n$ a fixed positive integer.

(a) Prove that $H = \{a \in G | a^n = e\}$ is a subgroup of $G$.

(b) Show by example that part (a) may be false if $G$ is nonabelian.

## Section 7.4

**6.** Prove that the function $h : \mathbb{Z}_8 \to \mathbb{Z}_8$ defined by $h(x) = 2x$ is a homomorphism that is neither injective nor surjective.

**12.** Prove that the function $h : \mathbb{R} \to GL(2, \mathbb{R})$ defined by $h(x) = \begin{pmatrix} 1 & 0 \\ x & 1 \end{pmatrix}$ is a injective homomorphism.

**14.** Prove that the additive group $\mathbb{Z}_6$ is isomorphic to the multiplicative group of nonzero elements in $\mathbb{Z}_7$.

**24.** Let $G$ be a multiplicative group. Let $G^{op}$ be the set $G$ equipped with a new operation $*$ defined by $a * b = ba$.

(a) Prove that $G^{op}$ is a group.

(b) Prove that $G \cong G^{op}$. [*Hint:* Corollary 7.6 may be helpful.]