Math 301 Homework 1 (due on August 30, 2023)

Appendix B

9. Let $A = \{1,\ 2,\ 3,\ 4\}$. Exhibit functions f and g from A to A such that $f \circ g \neq g \circ f$.

11. Is the subset B closed under the given operation?

(b) $B =$ odd integers; operation: addition in $\mathbb{Z}$.

(d) $B =$ odd integers; operation $*$ on $\mathbb{Z}$, where $a * b$ is defined to be the number $ab - (a + b) + 2$.

12. Find the image of the function f when

(b) $f : \mathbb{Z} \to \mathbb{Q}$; $f(x) = x - 1$.

(c) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = -x^2 + 1$.

24. Determine whether the given operation on $\mathbb{R}$ is commutative (that is, $a*b = b*a$ for all $a,\ b$) or associative (that is, $a * (b * c) = (a * b) * c$ for all $a,\ b,\ c$).

(a) $a * b = 2^{ab}$.

(b) $a * b = ab^2$.

(d) $a * b = (a + b)/2$.

(f) $a * b = b$.

25. Prove that the given function is injective.

(b) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = x^3$.

(d) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = -3x + 5$.

26. Prove that the given function is surjective.

(c) $f : \mathbb{R} \to \mathbb{R}$; $f(x) = -3x + 5$.

(d) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$; $f(a, b) = a/b$ when $b \neq 0$ and 0 when $b = 0$.

28.

(a) Let $f : B \to C$ and $g \to C \to D$ be functions such that $g \circ f$ is injective. Prove that f is injective.

(b) Give an example of the situation in part (a) in which g is not injective.

Section 1.1

2. Find the quotient q and remainder r when a is divided by b, without using technology. Check your answers.

(a) $a = -51$; $b = 6$.

(b) $a = 302$; $b = 19$.

9. Prove that the cube of any integer a has to be exactly one of these forms: $9k$ or $9k + 1$ or $9k + 8$ for some integer k. [*Hint:* By the Division Algorithm, a must be of the form $3q$ or $3q + 1$ or $3q + 2$.]