Gelfand-Kinillov dimension. Example GKdim (RXZ) = ? Recall: Fix a group homom $\alpha: \mathbb{Z} \rightarrow Aut(R)$ (α is determined by $\tau := \alpha(1)$, then $\alpha(n) = \sigma^n \forall n \in \mathbb{Z}$) Skew $R \neq \mathbb{Z} = R \neq \mathbb{Z} = \bigoplus_{n \in \mathbb{Z}} R \cdot t^n$ wita with multiplication $(rt^{n}) \cdot (st^{m}) = r\sigma(s)t^{n+m}$ $\forall r, s \in \mathbb{R}, n, m \in \mathbb{Z}.$

RXZ can be viewed as a quotient of an iterated skew polynomial ring: $\begin{array}{c} R \not\sim \mathbb{Z} \cong \mathbb{R}[t; \sigma, \sigma][t'; \hat{\sigma}], \sigma] \\ P^{f:} \\ \varphi: r t^{n} \longmapsto rt^{n}, n \in \mathbb{Z}, r \in \mathbb{R} \\ = t \end{array}$ Isomorphism of left R-modules & φ(rtⁿ st^m) = φ(rtⁿ)φ(st^m) => alg isomorphism. GK(R[t; 5,0][t'; 5',0])=2+6kdin(R) tt'-1 is a regular element of

R[t...][t...]: Say $P(t,t') = \sum_{k,e} T_{k,e} t^{k}(t)^{l}$, K, RER. p(t,t')(tt'-i) = 0. But Leading term (urt lexicographical order on {t^kt^p}) in LHS is the same as that of p(t,t') =>p(t,t')=0 By Thm last time,

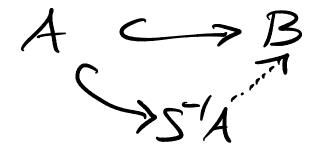
 $GKJim(R DZ) \leq 2 + GKJim(R) - 1$ = 1 + Gkdim(R).

Consider the injective algebra map. $R[t; 5, 0] \longrightarrow R \to \mathbb{Z}$ $\begin{array}{ccc}
211 & 211 \\
\bigoplus Rt^n \\
n=0
\end{array} \\
\begin{array}{c}
211 \\
\bigoplus Rt^n \\
\ll \mathbb{Z} \\
\end{array}$ rt" in rth ⇒ GKdim (R[t; 5,0]) ≤ GKdim (R>Z)

1) sEZ(A) (sa=as VaeA) Useis 2) VSES: 5 is left (= night) regular: $Sa = 0 \implies a = 0$. 3) ¥s, tes: s.tes; 1AES Define an equivalence relation on $S \times A$ by $(s,a) \sim (t,b)$ iff ta=sb $\begin{cases} s^{-1}a & s^{-1}a = t^{-1}b \\ ta = sb \end{cases}$ Define S'A = (S×A)/~ . Denote class

of (s,a) by s'a for sES, aEA. Check that D S'A is an algebra wrt operations (s'a)(t'b)=(st)'ab $(s^{-}a) + (t^{-}b) = (st)^{-}(ta+sb)$ 2) $A \rightarrow S'A$, $a \rightarrow 1'a$ 3) Image of any ses is invertible in S⁻A. 4) If PACAB is alg map st.

ang $\varphi(s)$ is invertible in B, then I comm. diagram



Theorem in the above setting GKdim (S'A) = GKdim (A) Pf. see[Lenagan - Krause, Ch.4]

Problem If R(G,t) is a GWA where t is a regular element of R. (Recall: tEZ(R) 303 always) $S := \left\langle \sigma^{n}(t) | n \in \mathbb{Z} \right\rangle_{Monorid}$ = $\left\{ \sigma^{n'}(t) \overline{\sigma^{n}}(t) \cdots \overline{\sigma^{n}}(t) \middle|_{k \ge 0} \right\}$ Show that $S^{-1}(R(\sigma,t)) \cong (S^{-1}R) \rtimes \mathbb{Z}$

Application: By Thm GKdim (R(5,t)) = GKdim S'(R(5,t)) By Problem = GKdim (S-R) NZ By Today = 1+GKdim (STR) By Thm = 1+GKdim(R) Aside $\varphi: A \rightarrow B$ surjective alg map. & suppose GKdim(A) = GKdim(B). Let I=ker q. BZA/I so I

can't contain any left or right regular elements. The $A = \bigcup_{n=0}^{\infty} A_n$ filtered alg. $\begin{pmatrix} 1 \in A_0 \subseteq A_1 \subseteq \dots, A_n A_m \subseteq A_{n+m} \\ gr A = \bigoplus_{n=0}^{\infty} A_n A_{n-1} & A_{-1} \cong 0 \end{pmatrix}$ Assume dim An < 00 Vnzo and that gr A is finitely generated Then GKdim (4) = GKdim (gr A).

Application $GKdim(U(Sl_2)) = GKdim(C[\bar{e},\bar{f},\bar{h}])$ More generally ¥fd Lie alg g: GKdim U(g) = GKdim gr U(g) $P^{BW}_{=}GKdim S(g)$ = dim g. Lsymm. algong